

Mathematics Pre-Course Work

1. Fractions

Examples: Calculate 1) $1\frac{2}{3} + 1\frac{1}{2}$

2) $\frac{3}{8} \times \frac{4}{9}$ and

3) $2\frac{1}{4} \div \frac{3}{5}$

Solution 1) $1\frac{2}{3} + 1\frac{1}{2}$

$$= \frac{5}{3} + \frac{3}{2}$$

$$= \frac{10}{6} + \frac{9}{6}$$

$$= \frac{19}{6}$$

$$= 3\frac{1}{6}$$

Solution 2) $\frac{3}{8} \times \frac{4}{9}$

$$= \frac{\cancel{3}^1}{8^2} \times \frac{4^1}{\cancel{9}_3}$$

$$= \frac{1}{2} \times \frac{1}{3}$$

$$= \frac{1}{6}$$

Solution 3) $2\frac{1}{4} \div \frac{3}{5}$

$$= \frac{9}{4} \div \frac{3}{5}$$

$$= \frac{\cancel{9}^3}{4} \times \frac{5}{\cancel{3}_1}$$

$$= \frac{3}{4} \times \frac{5}{1}$$

$$= \frac{15}{4}$$

Your Turn:

a) $\frac{3}{5} - \frac{1}{5} =$

b) $\frac{3}{7} - \frac{4}{7} =$

c) $\frac{2}{5} + \frac{3}{10} =$

d) $2\frac{1}{5} + \frac{3}{10} =$

e) $\frac{3}{8} - \frac{1}{6} =$

f) $1\frac{9}{10} - \frac{1}{3} =$

g) $\frac{2}{3} \times 3\frac{4}{5} =$

h) $2\frac{4}{5} \times 4\frac{5}{6} =$

i) $1 \div \frac{1}{5} =$

j) $\frac{2}{3} \div 1\frac{1}{9} =$

2. Laws of Indices

Examples: Simplify, writing as a single power. 1) $4^2 \times 4^5$

2) $4^9 \div 4^3$

3) $(5^2)^5$

Solution 1) $4^2 \times 4^5$
 $= 4^{2+5}$
 $= 4^7$

Solution 2) $4^9 \div 4^3$
 $= 4^{9-3}$
 $= 4^6$

Solution 3) $(5^2)^5$
 $= 5^{2 \times 5}$
 $= 5^{10}$

Your Turn:

a) $2^2 \times 2^5 =$

b) $3^4 \div 3^3 =$

c) $4^7 \div 4^3 \times 4^2 =$

d) $(x^5)^3 =$

e) $(a^4)^3 \div (a^2)^3 =$

f) $\frac{c^3 \times c^2}{c^7} =$

Examples: Calculate the value without a calculator. 1) 5^{-3}

2) $8^{1/3}$

Solution 1) 5^{-3}
 $= \frac{1}{5^3}$
 $= \frac{1}{125}$

Solution 2) $8^{1/3}$
 $= \sqrt[3]{8}$
 $= 2$

Your Turn:

g) $3^{-2} =$

k) $64^{\frac{1}{2}} =$

h) $7^0 =$

i) $2 \times 3^3 =$

l) $27^{\frac{1}{3}} =$

j) $2^2 \times 3^2 =$

3. Surds

Examples: Simplify, writing as a single surd where possible 1) $\sqrt{3} + 2\sqrt{3}$

Solution 1) $\sqrt{3} + 2\sqrt{3}$
 $= 3\sqrt{3}$

Solution 2) $\sqrt{2} \times \sqrt{5}$
 $= \sqrt{2 \times 5}$
 $= \sqrt{10}$

2) $\sqrt{2} \times \sqrt{5}$ 3) $\sqrt{90}$
Solution 3) $\sqrt{90}$
 $= \sqrt{9 \times 10}$
 $= \sqrt{9} \times \sqrt{10}$
 $= 3\sqrt{10}$

Your Turn:

a) $3\sqrt{7} + 2\sqrt{7} =$

d) $3\sqrt{6} \times \sqrt{6} =$

g) $\sqrt{54} =$

b) $4\sqrt{2} - 3\sqrt{2} =$

e) $\sqrt{18} =$

h) $\sqrt{12} =$

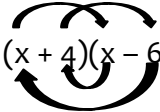
c) $3\sqrt{7} \times \sqrt{7} =$

f) $\sqrt{32} =$

4. Expanding and Simplifying Expressions

Examples: Expand and simplify 1) $4(x + 2) - 3(x - 1)$ and 2) $(x + 4)(x - 6)$

Solution 1) $4(x + 2) - 3(x - 1)$
 $= 4x + 8 - 3x + 3$
 $= 4x - 3x + 8 + 3$
 $= x + 11$

Solution 2) $(x + 4)(x - 6)$

 $= x^2 - 24 + 4x - 6x$
 $= x^2 - 2x - 24$

Your Turn:

a) $6b^2 + 5b - 1 + 3b + 4$

f) $(x - 1)(x + 1)$

k) $\frac{3x+6y}{3}$

b) $5(x - 3)$

g) $(3a + 2)(a - 1)$

l) $\frac{4}{2x+4}$

c) $-2(3x + 1)$

h) $(2b - 3)(3b - 2)$

m) $\frac{2}{5x-2}$

d) $3(4x + 2) + 5(2x - 1)$

i) $4(xy)$

e) $5(2x - 4) - 2(3x - 7)$

j) $(3x)^2$

5. Factorising

Examples: 1) $9xy + 15x$

2) $x^2 + 3x + 2$

3) $x^2 - 9$

Solution 1) $9xy + 15x$
 $3x(\quad)$
 $3x(3y + 5)$

write the highest common factor, HCF, outside the brackets
divide both parts of the expression by the HCF
check your answer by multiplying through the brackets.

Solution 2) $x^2 + 3x + 2$
 $(x \quad)(x \quad)$
 $(x + 2)(x + 1)$

Set out double brackets, writing an x in each one
 Think of two factors of 2 that will add to 3.

Solution 3) $x^2 - 9$
 $(x + 3)(x - 3)$

9 and x^2 are both square numbers; this is a DOTS question!
 add and subtract the square roots in the brackets.

Your Turn:

- | | | |
|--------------------|---------------------|--------------------|
| a) $4x + 8 =$ | e) $x^2 + 4x + 3$ | i) $x^2 - 36$ |
| b) $2ab + ad$ | f) $x^2 + 8x + 15$ | j) $2x^2 + 7x + 5$ |
| c) $8x^2 - 10x$ | g) $x^2 + 12x - 28$ | |
| d) $8ab^2 - 4a^2b$ | h) $x^2 - 17x + 30$ | |

6. Solving Linear Equations

Examples:

Solve the following equations 1) $5x + 4 = 11$ and 2) $7(x - 2) = 7$

Solution 1)
 $5x + 4 = 11$
 $5x = 11 - 4$
 $5x = 7$
 $x = 7 \div 5 = \frac{7}{5}$

Solution 2)
 $7(x - 2) = 7$
 $7x - 14 = 7$
 $7x = 7 + 14 = 21$
 $x = 21 \div 7 = 3$

Your Turn:

- | | | |
|----------------------|----------------------------|---|
| a) $5x + 7 = 32$ | d) $3p + 2 = 5 - p$ | f) $\frac{3x-13}{7} + \frac{11-4x}{3} = 0$ |
| b) $2(2x - 7) = 7$ | e) $2 - 3(2x - 5) = 7 - x$ | g) $\frac{6}{x} + \frac{3}{2x} = \frac{5}{2}$ |
| c) $4x - 5 = 2x + 7$ | | |

7. Formulae

Examples: Substitute into the following formulae to determine the missing value 1) If $x = ab - c$, find x when $a = 4$, $b = \frac{1}{2}$ and $c = -5$

$x = ab - c$
 $= 4 \times \frac{1}{2} - (-5)$ $4 \times \frac{1}{2} = 2$ and $-(-5)$ is the same as $+5$
 $= 2 + 5$
 $= 7$

Your Turn:

- a) $x = ab + c$ Find x when $a = \frac{2}{3}$, $b = 9$ and $c = -3$
- b) $x = 2a^2$ Find x when $a = \frac{3}{4}$

- c) $A = 4\pi r^2$ Find r when $A = 616$
 d) $a = b - \frac{1}{2}c$ Find c when $a = 6$ and $b = 10$
 e) $v = u + at$ Find a when $v = 21.5$, $u = 4$ and $t = 7$

Examples: Make x the subject of each of these formulae;

1) $a = x - ab$ 2) $xy = w$ and 3) $f = d(x + e)$

Solution 1)

$a = x - ab$ Treat ab as a single item; add ab to each side
 $a + ab = x$ Swap each side to give $x =$
 $x = a + ab$

Solution 2)

$xy = w$ Remember that $xy = x \times y$, you need to divide by y
 $x = \frac{w}{y}$

Solution 3)

$f = d(x + e)$ Firstly multiply out the brackets
 $f = dx + de$ Treating de as a single item; subtract de from each side
 $f - de = dx$ Divide by d
 $\frac{f - de}{d} = x$ Swap each side to give $x =$
 $x = \frac{f - de}{d}$

Your Turn:

- f) $3x = b$ i) $2(3x - 1) = 5y$ l) $\sqrt{x - 2} = y$
 g) $\frac{x}{5} = d$ j) $ax = bx + c$
 h) $f = 4 - x$ k) $mx = u - 2x$

8. Solving Quadratic Equations

Examples:

Solve the following quadratic equations; 1) $x^2 - 8x + 12 = 0$ and 2) $y^2 + 13y + 40 = 0$.

Solution 1)

In the quadratic equation $x^2 - 8x + 12 = 0$, the expression can be factorised. So $(x - 6)(x - 2) = 0$
 We set each factor pair equal to zero to get our two solutions.
 $x - 6 = 0$ and $x - 2 = 0$

$$x = 6 \text{ or } 2$$

Solution 2)

In the equation $y^2 + 13y + 40 = 0$, we have $a = 1$, $b = 13$ and $c = 40$. So

$$y = \frac{-13 \pm \sqrt{13^2 - 4 \times 1 \times 40}}{2 \times 1} = \frac{-13 \pm \sqrt{169 - 160}}{2} = \frac{-13 \pm \sqrt{9}}{2} = \frac{-13 \pm 3}{2} = \frac{-13 + 3}{2} \text{ or } \frac{-13 - 3}{2} = -5 \text{ or } -8$$

Your Turn:

a) $n^2 + 5n + 4 = 0$

d) $x^2 - 2x - 6 = 0$

b) $t^2 - 4t - 12 = 0$

e) $x^2 - 6x - 8 = 0$

c) $x^2 - 81 = 0$

f) $3x^2 + 10x - 7 = 0$

9. Simultaneous Linear Equations

Examples:

Solve the following pairs of simultaneous equations

1) $7x + 2y = 32$
 $x + y = 1$

2) $5x + 2y = 26$
 $4x - 3y = 7$

Solution 1)

Double the second equation to give

$$7x + 2y = 32$$

$$2x + 2y = 2$$

Subtract the new second equation from the new first, and solve the resulting equation to find x

$$5x = 30$$

$$x = 6$$

Substitute into either of the original equations to find y

$$x + y = 1$$

$$\Rightarrow 6 + y = 1$$

$$y = -5$$

Solution 2)

Multiply the first equation by 3 and the second equation by 2 to give

$$15x + 6y = 78$$

$$8x - 6y = 14$$

Add the two equations and solve

$$23x = 92$$

$$x = 4$$

Substitute into either of the original equations to find y

$$5x + 2y = 26$$

$$\Rightarrow 20 + 2y = 26$$

$$2y = 6$$

$$y = 3$$

Your Turn:

a) $5x - 3y = 23$

$$2x + 3y = 26$$

b) $y = 2x + 1$

$$3y + 10x = 7$$

c) $5x + 2y = 11$

$$3x + 7y = -5$$

d) $x + 2y = 4$

$$2x + y = 5$$

e) $3x - 6y = 33$

$$x - 3y = 16$$

10. Straight Line Graphs

Draw the graph and state the gradient and y-intercept for each line.

Example: $y = 3x - 2$

Either set up a table of values to get some coordinates or go straight to the graph using the gradient and y-intercept.

x	-2	0	2
y	-8	-2	4

$$y = 3 \times -2 - 2 = -8$$

$$y = 3 \times 0 - 2 = -2$$

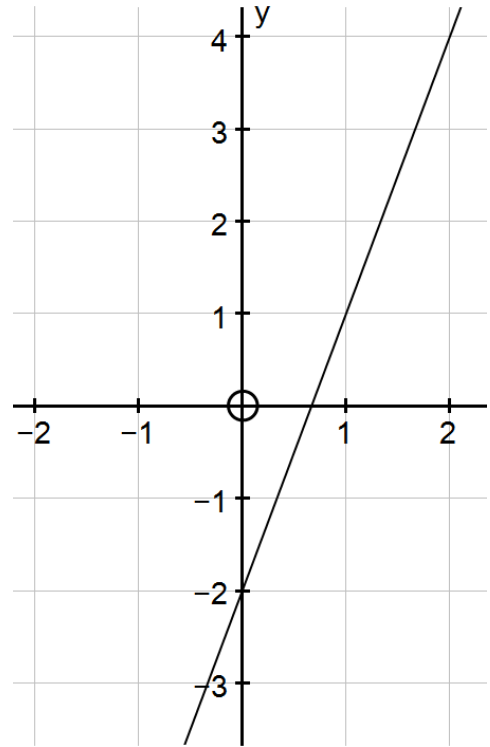
$$y = 3 \times 2 - 2 = 4$$

When written in the form $y = mx + c$

$$m = \text{gradient} = \frac{\text{change in } y}{\text{change in } x}$$

$c =$ y-intercept

for this equation, $m = 3$ and $c = -2$



Your Turn:

a) $y = 2x + 1$

b) $y = \frac{2}{3}x - 3$

c) $x + 2y = 6$